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Use of the List Viterbi Algorithm to Compute the Distance Spectrum of Trellis Codes and ISI Channels

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Abstract — We propose to compute the distance spectrum of arbitrary trellis codes (including convolutional codes, trellis-coded modulation, continuous phase modulation, etc.) and intersymbol-interference (ISI) channels by means of a modified list Viterbi algorithm (LVA). This search procedure is (i) computationally efficient, (ii) is applicable to linear as well as non-linear codes, (iii) can be applied to arbitrary distance measures, (iv) can be used for MLSE as well as RSSE or related techniques, and (v) guarantees that an ordered list of the N nearest error paths is produced. A sample results illustrates the distance spectra of linear ISI channels, both for MLSE and ideal RSSE receivers.

I. INTRODUCTION

Prior solutions to compute the free distance of nonlinear codes include sequential algorithms, the Viterbi algorithm, and the Dijkstra algorithm. Solutions to compute the distance spectrum include sequential algorithms, transfer function methods, and a modified Viterbi algorithm with state-splitting and multiple passes, among other techniques. For special applications and particularly in the case of linear codes extensive simplifications are possible.

In the present paper, we propose to apply a modified LVA for the purpose of computing the distance spectrum of arbitrary trellises. LVAs compute an ordered list of the N best paths. Serial and parallel LVAs have extensively been investigated in [1] and the references therein in the context of decoding and related applications, but, to our best knowledge, not for distance calculations.

II. DISTANCE CALCULATION USING AN LVA

Throughout this paper, we assume the existence of a trellis with a finite number of states. We consider a linear code first. In order to compute the distance spectrum with N error paths, it is sufficient to design a modified LVA for the original trellis taking the N best survivors into account, and to apply this LVA given noise-free channel outputs. An ordered list of the N nearest error paths is produced, if the following modifications are done:

1. All error paths taken into account must diverge from the transmitted sequence at time k = 0 and re-merge at time k' > 0. All other paths must be excluded, particularly the ML path and all paths that diverge more than once from the transmitted path.

2. Instead of outputting the N most likely information sequences [1], we output the accumulated path metrics (i.e., the distances) and the corresponding path weight (multiplicity) a_d and/or information weight a_d.

Without loss of generality, the transmitted sequence may be the all-zero sequence. Then, the all-zero path (i.e., the ML path) may be eliminated by setting the N accumulated distances of the all-zero state at the second interval of the trellis to infinity. The number of spectral lines is less than the number of error paths N actually computed, when the multiplicity a_d > 1 for at least one distance d. Whether a serial or a parallel type of LVA should be accomplished depends on memory and complexity constraints, among others. If the trellis is of finite length, the LVA may operate on the full trellis, otherwise a stop criterion must be applied.

These general design criteria also hold for nonlinear codes, which are discussed next. In the general case, we design the LVA to operate on the product trellis in order to take all error events into account. If the error events depend on difference symbols only, we may use the difference trellis instead. This is the case of linear ISI channels and CPM, e.g. In any case the symmetry of the error states has to be taken into account, either by eliminating redundant error events or by reducing the number of states.

For illustration, consider a time-invariant linear channel with binary inputs a_t ∈ {±1} and channel coefficients h_k, 0 ≤ k ≤ L. The difference symbols, \( d_k = a_k - \hat{a}_k \), take the values \( \{-2, 0, +2\} \). In case of MLSE, the difference trellis has \( 3^L \) states, whereas the original trellis has \( 2^L \) states. However, due to the symmetry of the error states, we can use an equivalent difference trellis that has only \( (3^L + 1)/2 \) states. Without loss of generality, we may assume that the all-zero difference sequence has been transmitted. A new spectral line is computed whenever an error path re-merges the all-zero difference path. In case of reduced-state sequence estimation (RSSE), the original trellis has \( 2^K \) states, whereas \( 0 < K < L \). A new spectral line is computed whenever an error path merges in one of the \( (3^K + 1)/2 \) hyper states. Otherwise, the search algorithm is the same as described above.

Fig. 1 shows the (truncated) distance spectrum for an ISI channel given MLSE and ideal RSSE. (Ideal RSSE does not take error propagation into account.) The information weight is moderate for error paths with small distance, whereas larger spectral lines have a larger multiplicity and are less spread out.

![Fig. 1: Distance spectrum for MLSE and ideal RSSE for a binary, linear, time-invariant ISI channel with 32 states.](image)

REFERENCE