RELIABILITY-BASED HARQ USING WORD ERROR PROBABILITIES

Justus Christian Fricke, Hendrik Schoeneich, and Peter Adam Hoeher

Faculty of Engineering, University of Kiel
Kaiserstr. 2, 24143 Kiel, Germany
phone: +49 431 880 6126, fax: +49 431 880 6128, email: {jf, hs, ph}@tf.uni-kiel.de
web: www-ict.tf.uni-kiel.de

ABSTRACT
Reliability-based hybrid automatic repeat request (RB-HARQ) protocols utilise reliability information from a soft-output channel decoder to improve the reliability of digital communication systems with feedback. In this work, we derive a technique to approximate the word error probability given log-likelihood ratios provided for the information bits. No error detecting code is needed when using the word error probability as a reliability criterion for retransmissions. Furthermore, we show that this estimate can be improved by applying subblock-by-subblock decoding. The word error probability can be used as a reliability measure for ARQ protocols to decide whether a frame or codeword can be considered reliable enough to be accepted, or further transmissions are needed. To our best knowledge, the presented methods to calculate word error probabilities have never been used as a reliability criterion in RBHARQ protocols. Another possible application are Monte Carlo simulations of word error rates in coded transmissions.

1. INTRODUCTION
Many modern data communication systems make use of hybrid automatic repeat request (HARQ) schemes, which combine forward error correction (FEC) and automatic repeat request (ARQ) protocols. While the FEC corrects as many errors as possible, ARQ initiates a retransmission of a code word or transmission of extra redundancy if residual errors are detected at the receiver side. HARQ protocols normally result in higher throughput than pure ARQ protocols as less retransmissions are needed due to the error correcting capability of the FEC. In recent years another interesting advantage of HARQ protocols attracted interest. As soft-output decoding is becoming increasingly popular due to the possibility to apply iterative decoding, reliability information of the transmitted information bits is available in many receivers. ARQ protocols which utilise this reliability information are called reliability-based ARQ (RBARQ) protocols.

So far three different techniques to utilise the reliability information provided by a soft-output channel decoder are known from literature:

1. Soft outputs in form of log-likelihood ratios (LLRs) from independent observations (e.g., different transmissions) can be added to form more reliable estimates. This can be considered as an elegant form of Chase combining [1] and was proposed in [2] in combination with iterative processing.

2. As the magnitude of a log-likelihood value is directly connected to the error probability of the corresponding bit, it can be used to determine which bits most likely caused a word error. Using this knowledge, the number of bits that have to be retransmitted to allow for quasi error-free decoding can be reduced by retransmitting only the unreliable bits [3], [4], and [5].

3. The reliability information is used to estimate the error probability of the transmitted data bits to ensure a required quality of service (QoS) and/or to optimise the transmit power or the effective code rate. Such an approach is presented in [6] for different bit-error-rate based quality criteria while the authors of [7] and [8] derived a way to estimate the word error rate based on the LLRs.

The focus of this paper is on the latter technique. The advantage of reliability-based retransmission criteria is that no error detecting code is necessary. Moreover, it is also more flexible than a simple correct/wrong decision for a received word, because the decision for a retransmission can be based on the required QoS in terms of the word error rate (WER). Instead of discarding all wrong frames, frames which are reliable enough (for a maximum allowed WER) are accepted which results in increased throughputs.

By means of simulations, the authors of [5] found a criterion in terms of a minimum bit reliability to ensure a certain maximum word error probability. In [8], the average of the magnitude of the soft outputs is calculated to obtain a measure for the word reliability. This word reliability is afterwards mapped to the simulated word error rate to obtain a reliability criterion. Similar to [8], we calculate the reliability for the decoded word in this paper. However, we directly calculate the word error probability from the soft outputs, i.e., a subsequent mapping of the calculated reliability measure to the WER by means of simulations is not necessary anymore. While we concentrate on convolutional codes, the methods and conclusions are generic and can be applied to arbitrary codes.

The remainder of this paper is organised as follows. In Section 2, a short description of the system model and a brief explanation of the concept of a RBHARQ scheme is given. The optimal subblock-by-subblock decoder [9] is introduced in Section 3. In Section 4, the novel approach to calculate the word error probability based on the soft channel decoder outputs is explained. In Section 5, numerical results showing the feasibility
of our approach and its application in a simple ARQ scheme are provided. Soft Monte Carlo word error probability simulations are a possible application, as briefly explained in Section 6. Conclusions are drawn in Section 7.

2. SYSTEM MODEL

A system model widely used in literature is illustrated in Fig. 1. A sequence of information bits $u_k$ is stored in a transmit buffer (if ARQ is applied) and encoded by an FEC encoder. The code bits in the code word $c$ are mapped to a symbol alphabet and are then transmitted over the channel. At the receiver, the sequence of received samples $y$ is demapped and decoded. For conceptual purposes, we assume that the detector is able to compute true log-likelihood ratios

$$\hat{u}_k = \log \frac{P(u_k = +1|y)}{P(u_k = -1|y)}$$

(1)

for the transmitted info bits. Throughout this paper, a tilde denotes a log-likelihood value while the hat denotes a hard decision, i.e., $\hat{u}_k \in \{-1,+1\}$ and $\hat{u}_k \in \mathbb{R}$. The index $k = 1,...,K$ denotes the position of a bit in the info word, where $K$ denotes the word length.

Figure 1: A simple transmission system using FEC and ARQ. The decoder is providing soft outputs $\hat{u}_k$ in form of LLRs.

3. OPTIMAL SUBBLOCK-BY-SUBBLOCK DECODING

A well-known decoder, which is able to provide a posteriori symbol probabilities, is the optimal symbol-by-symbol decoder (OSSD) [10], also known as the BCJR algorithm. A less familiar decoder is the optimal subblock-by-subblock decoder (OBBD) [9]. The OBBD is a generalisation of the OSSD that calculates joint a posteriori probabilities of $N$ successive symbols instead of the marginal symbol probabilities. As we will see in the following, this property is advantageous for our purpose. For a subblock length of $N = 1$ the OBBD corresponds to the OSSD since one subblock corresponds to one symbol.

For the decoding done by the OBBD the word $u$ is divided into subblocks $u_i^{i+N-1}$ of length $N$ with $i = 0, N, 2N, ..., K + N - 1$. The a posteriori subblock probabilities are denoted as $P(u_i^{i+N-1}|y)$.

It should be noted that the complexity of the OBBD is even lower than that of the OSSD if $N$ is less or equal to the memory length of the code as the subblock probabilities are implicitly calculated by the forward and backward recursion of the OSSD. For subblock lengths greater than $N$ the decoding complexity is increasing dramatically since the number of subblock hypotheses is equal to $2^N$. For $N = K$ the OBBD performs a complete search over all possible words. We refer the interested reader to [9] for further details.

If soft outputs for the info symbols are needed, the symbol probabilities $P(u_k)$ are calculated as the marginal probabilities by summing up the probabilities of all $2^{N-1}$ subblock hypotheses $\tilde{u}_i^{i+N-1}$ which contain the $(i+n)$th bit being ‘+1’ or ‘-1’

$$P(u_{i+n} = +1|y) = \sum_{\tilde{u}_i^{i+N-1}} P(u_i^{i+N-1}|y),$$

$S_{+1}$ being the set of all $u_i^{i+N-1}$ whose $n$th bit is a ‘+1’ and

$$P(u_{i+n} = -1|y) = \sum_{\tilde{u}_i^{i+N-1}} P(u_i^{i+N-1}|y),$$

$S_{-1}$ being the set of all $u_i^{i+N-1}$ whose $n$th bit is a ‘-1’.

The LLR $\tilde{u}_{i+n}$ can be computed according to (1).

4. CODEWORD RELIABILITY

The exact bit error probability for $u_k$ can be calculated using the magnitude of the corresponding log-likelihood ratio

$$P_{b,k} = P(\hat{u}_k \neq u_k) = \frac{1}{1 + e^{[\tilde{u}_k]}}$$

(2)

see for example [11]. The average bit error probability $P_b$ of the whole word can be obtained as

$$P_b = \frac{1}{K} \sum_{k=1}^{K} P_{b,k}.$$

We now calculate the probability $P_w$ that the received word is not correctly decoded. In [7] and [8], $P_w$ has been estimated based on the average magnitude of the LLRs, $\frac{1}{N} \sum_{k=1}^{K} |\tilde{u}_k|$, and a subsequent mapping. This mapping function highly depends on the channel code, the symbol alphabet, the channel, and the receiver structure and must therefore be determined for each possible combination of these components. We follow a generic approach instead, which needs no mapping function and is therefore independent of these components. Let $\overline{P}_w$ be the probability that the word is decoded correctly. Then the word error probability (WEP) is simply

$$P_w = 1 - \overline{P}_w = 1 - P(\hat{u} = u)$$

(3)

as the set of all possible info words is the set union of the correct word and the set of all wrong words. Using (2) and (3), an approximation of $P_w$ can be calculated as follows:

$$\hat{P}_w = 1 - \prod_{k=1}^{K} (1 - P_{b,k})$$

$$= 1 - \prod_{k=1}^{K} \left(1 - \frac{1}{1 + e^{[\tilde{u}_k]}}\right)$$

$$= 1 - \prod_{k=1}^{K} \frac{1}{1 + e^{-|\tilde{u}_k|}}.$$
If the soft outputs are conditionally independent, (4) is exact. That would be the case for uncoded transmission over an AWGN channel or coded transmission applying a repetition code. However, the soft outputs are usually correlated. For convolutional codes the correlation of bits is highest for neighbouring bits and is decreasing with increasing distance. If instead of the bit error probabilities $P_{b,k}$ in (4) subblock error probabilities

$$P_{s,i} = 1 - P(\hat{u}_i^{i+N-1} = u_i^{i+N-1})$$

are used, we get

$$P_w \approx \hat{P}_{w,N} = 1 - \prod_{i=0}^{K-N-1} (1 - P_{s,i}) = 1 - e^{-\sum_{i=0}^{K-N-1} |\hat{u}_i - u_i|}.$$  

(6)

Using the subblock error probabilities to calculate an estimate of the WEP, the correlation of neighbouring symbols inside one subblock is taken account. This results in a better approximation of $P_w$ since the correlation is highest between neighbouring symbols. If the subblock length $N$ is equal to 1, $P_{s,i} = P_{b,k}$ and therefore (4) is equal to (6). For $N = K$ the subblock-by-subblock decoder provides the word probabilities, $P_w = P_{w,N=N=K}$, and (6) is exact.

If $K$ is large it may be advantageous for numerical reasons to avoid the product in (4). The product can be substituted by a sum if the calculation of $\hat{P}_w$ is done in the log domain. Then $\hat{P}_w$ becomes

$$\log \hat{P}_w = \log \prod_{k=1}^{K} (1 - P_{b,k})$$

$$= \sum_{k=1}^{K} \log \left(1 - P_{b,k}\right)$$

$$= - \sum_{k=1}^{K} \log(1 + e^{-|\tilde{u}_k|})$$

$$\Rightarrow P_w = 1 - e^{\log \hat{P}_w}.$$ 

Note that this simple way to calculate the error probability of a decoded word given the LLRs provided by a soft-output channel decoder does not depend on the distribution of the LLRs. The code word error probability can be calculated in a similar way by replacing the info bit error probabilities by the code bit error probabilities.

To our best knowledge, this method of calculating the WEP directly from the LLRs has never been suggested as a reliability criterion. Compared to other reliability to word error probability mappings, e.g., in [5] and [8], there are no different mappings or reliable thresholds for different signal-to-noise ratios, channel models, channel codes, or word lengths necessary as $P_w$ is the actual word error rate if the above constraints are fulfilled. For iterative decoding it is also necessary that the conditional dependencies of the soft outputs are low, which legitimates (4).

5. NUMERICAL RESULTS

The simulation results presented in this section are obtained by Monte Carlo simulations of transmissions via an AWGN channel. The signal-to-noise ratio (SNR) is averaged by the number of code bits of one transmission to obtain the average SNR per info bit. For every SNR point the word error rate was calculated from a number of word transmissions that contains at least 200 erroneous decoded words with an info word length of $K = 192$. The ARQ protocol used for the simulations is a simple stop-and-wait ARQ (SW-ARQ) protocol [12] with an infinite number of retransmissions. A predefined maximum allowed target $P_{w,t}$, $P_{w,t}$, is given as a threshold. A packet is accepted if its WEP according to (6) is lower than or equal to $P_{w,t}$. WER/WEP results are calculated only from accepted packets, i.e., the residual WER after application of the ARQ is shown. Recall that accepted packets are not necessarily error-free.

5.1 Word Error Probability as a Reliability Criterion

We now show the usability of the results in Sec. 4, i.e. the calculation of word error probabilities according to (4) and (6). For that reason, we compare the $P_{w,N}$ calculated according to (6) and averaged over multiple word transmissions to the relative number of erroneously decoded words, i.e., the WER based on Monte Carlo simulation. Note that the latter method to estimate the average WEP requires perfect knowledge about the transmitted words which is not the case for the reliability-based method of (4). Figure 2 compares the results for different SNRs.

From Fig. 2 is visible that their is always a gap between the averaged $P_w$, $P_{w,N}$, and the WER. Fortunately, $P_w$, $P_{w,N}$, is pessimistic and therefore results in a safe decision when used as a retransmission criterion. Furthermore, the difference between the averaged $P_w$, $P_{w,N}$, and the WER is always smaller than 0.5 dB.

![Figure 2: Average $\hat{P}_{w,N}$/WER of different convolutional codes (memory length of 4, 6, and 8) using a subblock-by-subblock decoder with subblock lengths of $N = 1$ and $N$ equal to the memory length of the code.](image-url)
the memory length compared to a subblock length of 1. Here we use subblock lengths equal to the memory lengths because an even longer length would increase the complexity dramatically compared to the OSSD while offering only marginal improvements. However, subblock lengths up to the memory length do not increase the problem complexity (cf. Sec. 3). For a subblock length equal to the word length the averaged $\hat{P}_{w,N}$ is equal to the WER.

5.2 Word Error Probability in Reliability-Based ARQ

We now implement a simple type-I HARQ protocol that employs no packet combining but just retransmits an unreliable word until the word error probability estimate $\hat{P}_w$ of the received word is equal or lower than $P_{w,t}$. For the throughput simulations the idle time is assumed to be zero as we are concentrating on the impact of the word reliability. The throughput is defined as

$$\eta = \frac{\text{number of accepted info bits}}{\text{number of transmitted code bits}}.$$  (7)

The ARQ scheme used in this model is the simple stop-and-wait (SW) ARQ protocol.

Fig. 3 shows the resulting residual word error rates for a (23, 35) convolutional code when employing this protocol. It can be observed that the given thresholds are always fulfilled in terms of resulting word error rates, which are always below $P_{w,t}$. Before the start of the plots, no transmitted word fulfilled the maximum WEP criterion.

From the corresponding throughput curves in Fig. 4 it can be seen that the throughput increases if $P_{w,t}$ is higher. If a user or application allows for higher packet error probabilities, i.e., also accepts less reliable packets, the throughput is higher. This effect may be used by adaptive ARQ schemes, which enable a trade-off between data rate and word error rate.

Fig. 5 shows the impact of reliability-based bit-wise retransmissions as described in [3] on the reliability criterion. If the WEP is too low, the 20 code bits with the lowest reliability are retransmitted. As the LLRs from different transmissions may be added to form a more reliable soft estimate of a code bit, the reliability criterion still holds and the WEP still corresponds to the WER. The effect of the retransmissions can be seen especially for an SNR below 2 dB where many extra transmissions are needed to form a reliable packet. The combined packet usually has a WEP/WER lower than $P_{w,t}$. Since at the SNR points beyond 2 dB many of the packets are accepted after the first transmission, the WER resembles the results without incremental redundancy in Fig. 3.

6. FURTHER APPLICATIONS OF THE WORD ERROR PROBABILITY

In [11] the idea to use LLRs in Monte Carlo simulations was introduced. It was proposed to calculate the average bit error probability by averaging the bit error probabilities of the single bits instead of counting the occurring errors. These “soft” simulations have the
advantage that they usually require much shorter simulation times to achieve reliable BER estimates.

For Monte Carlo simulations of HARQ protocols the required number of simulated packet transmission to get reliable packet/word error estimates is usually quite high because of the low error probabilities. From Fig. 2 it can be seen that the average of the calculated word error probability according to (4) is not far away from the simulated word error rate. A reliable estimate of the average word error probability can be obtained by much fewer simulated transmissions than compared to the case where the word error rate is simulated directly. Given $P_{w,N}$ based on the log-likelihood ratios of the bits in Sec. 4 a soft estimate of the word error rate and the throughput can be obtained by relatively small number of simulated transmissions. Taking into account that the estimates of the WEP are pessimistic in average, the resulting soft-simulated throughput estimates is slightly lower than that of conventional simulations.

7. CONCLUSIONS

In this paper a reliability criterion based on the WEP for use in RBHARQ protocols is investigated. It involves the calculation of the approximated word error probability from the reliability information of the information bits which are provided by a soft-output channel decoder. Furthermore, we showed that the estimate of the WEP can be improved by applying optimal subblock-by-subblock decoding [9] without increasing the complexity. We demonstrated the applicability and practicability of our reliability criterion by numerical results employing a simple ARQ protocol. It may be used as a criterion whether a packet should be retransmitted as well as for adaptation strategies as proposed in [8], [13], which allow a trade off between achievable throughput and word error rate. The reliability criterion can also replace an error detecting code and safe the code bits needed for the error detection.

Since the word error probability only depends on the soft outputs and its calculation stays the same for different scenarios as different channel codes, decoders, SNRs, and channels, it has a clear advantage over other WEP estimating criterions proposed before. It can be easily applied in an ARQ protocol without requiring knowledge of the underlying physical channel or other physical layer parameters, except for the knowledge of the channel decoder soft outputs.

Another application are Monte Carlo simulations of the WEP similar to the BEP simulations described in [11].

REFERENCES