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On Channel Coding and Multiuser Detection for DS–CDMA

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Abstract

Two issues concerning direct–sequence code–division multiple access (DS–CDMA) multiuser detection are addressed: Reliability estimation (i.e., soft–output detection), and a complexity reduction technique to simplify the optimum (maximum–likelihood) multiuser detector. The soft–outputs are not only suitable for improving the performance of a next stage (in this study an outer soft–decision Viterbi decoder), but also for reducing the complexity of the multiuser detector itself. In this way, a family of suboptimum multiuser detectors with adjustable performance/complexity tradeoff is obtained in a structured manner from the optimum multiuser detector.

Reliability generation is applied by post–processing hard decisions of an auxiliary (single or multiuser) detector delivering tentative decisions given matched filter outputs. Reduced–complexity multiuser detection is based on exhaustively searching only in a subset of unreliable decisions given the reliability estimates. A decision–feedback mechanism is shown to be included.

In particular it is indicated that the decision–feedback multiuser detector is outperformed on a Gaussian channel, and that almost all multiuser interference is cancelled out in a coded multiuser system in Rayleigh fading, i.e., in a severe near–far situation. The latter result corresponds to a gain on the order of 4 dB at a bit error rate of 10−2 compared to conventional detection, or a loss of only about 1 dB compared to a single user system.

1 Introduction

Multiuser detection for DS–CDMA channels and equalization of (single–user) intersymbol–interference channels are related problems. Both channels can be described by a discrete–time system model [8, 5, 6]. Hence, it is not surprising that (some) multiuser detectors have counterparts with equalizers. The “decorrelation multiuser detector (decorrelator)” [7, 8], the “decision–feedback (DF) multiuser detector” [5, 6], and the “optimum multiuser detector” [1, 2, 3] are the counterparts of the zero–forcing linear equalizer, the decision–feedback equalizer, and the maximum–likelihood equalizer, respectively.

Most publications on DS–CDMA multiuser detection are considering uncoded transmission so far, with a few exceptions such as [9]. Coded DS–CDMA systems, however, are of potential practical interest [10]. Consider the uplink, which is the critical link. Since the base station has full knowledge about the channel codes, spreading sequences, bit rates, etc., of its K users, combined multiuser detection/channel decoding would be optimal. Combined multiuser detection/channel decoding, however, is unfeasible if the number of users is moderate or large (remember that the complexity of the optimum multiuser detector grows exponentially with K even in the uncoded case), and if interleaving is performed (which is assumed here). Therefore, we propose in the present paper to perform multiuser detection first, but to deliver reliability information to enhance the channel decoder. One objective of this paper is to derive a suitable soft–output multiuser detector. To obtain this goal, a low–cost version of the recently proposed post–processing likelihood detector [11] is applied. Another objective is to derive suboptimum low–complexity multiuser detectors in a straightforward manner from the optimum multiuser detector.

Consider for the moment an uncoded synchronous direct–sequence code–division multiple–access system with K users. Given full knowledge of the signaling waveforms and the path attenuations (and delays in the asynchronous case) for all channels k, 1 ≤ k ≤ K, the maximum–likelihood sequence detector (MLSD) is known to select the information sequence b = (b1, . . . , bK)H that maximizes the log–likelihood function

\[ L(b) = 2R[bHy] - bHCb, \]

where \( y = (y_1, . . . , y_K)^H \) is the output of the bank of matched filters sampled at symbol rate T, \( R = (r_{k,k}') \) is the cross–correlation matrix which depends on the signaling waveforms, \( C = \text{diag}(c_1, . . . , c_K) \) is the path attenuation matrix, and \( H \) denotes the conjugate transpose [1, 2, 3, 7].

\[ \hat{b} = \arg \max_b L(b), \]

where \( y_k = \int_0^T r(t)c_k a_k(t) \, dt \) with \( r(t) = \sum_{k=1}^K b_k c_k a_k(t) + n(t) \) being the received signal, \( a_k(t) \) the normalized spreading

The structure of a combined multiuser detector/channel decoder is similar to a combined equalizer/channel decoder, and uses the fact that not all coded information sequences are allowed.

Channel coding is optional if we assume i.i.d. channel bits due to sufficient bit interleaving.

For the case of presentation our emphasis here is on synchronous transmission, i.e., all users are assumed to be chip and symbol synchronized. An observation interval of one symbol slot T is sufficient to investigate. This assumption does not effect the main message of this paper. The asynchronous case is briefly discussed in Section 5.

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waveforms in $t = [0, T]$, and $n(t)$ an additive white Gaussian noise process.

Maximization of (1) combines two tasks: Likelihood evaluation and ML sequence detection. Dynamic programming by means of a forward recursion (e.g., Viterbi algorithm), backward recursion, or forward–backward recursion (e.g., optimum symbol–by–symbol detection) is possible, which, however, is only feasible for a moderate number of channels, $K$, from a practical point of view.

The first observation motivating this paper is that likelihood evaluation and MLSD can be separated: Instead of combining these tasks, we propose to feed precomputed hard decisions into the likelihood function and to vary only a certain subset of possible permutations of information bits. The precomputation can be done, in principle, by any single or multiuser detector, which we will call auxiliary detector in the following because we treat its hard decisions only as tentative decisions.

The second observation motivating this paper is that an unreliable decision $b_k$ in channel $k$ affects a similar likelihood value when this information bit would be flipped (assumed binary transmission). Hence, a reliability indicator is obtained by computing the likelihood difference by successively flipping only the $k$–th bit, $1 \leq k \leq K$, given the tentative information bits of the remaining $K-1$ channels. The complexity of this algorithm is $O(K)$, i.e. linear in $K$. It is the simplest realization of a post–processing likelihood processor recently proposed by the author [11]. It is shown that a decision–feedback mechanism is included, and that this algorithm is conceptually simpler and (at least for the example under test) slightly more efficient than the decision–feedback (DF) multiuser detector derived in [5, 6]. Generalizations are straightforward.

Finally, the third observation is that the bit error rate (BER) is dominated by weak decisions, i.e., those channels with a small reliability. We use this effect by running the MLSD only for a few unreliable channels $k$, whereas the other channels are determined by the auxiliary detector. Hence, a reduced–complexity multiuser detector with adjustable performance/complexity tradeoff is obtained.

The considered scenario is a synchronous DS–CDMA system with Gold spreading sequences [4, 5], optionally convolutionally coded. An important result will be that almost all multiuser interference is cancelled. The loss is only about 1 dB, which implies that a complicated combined multiuser detector/channel decoder is not needed.

2 Reliability Generation by Post–Processing Hard Decisions

2.1 Derivation of an $O(K)$ Likelihood Estimator

Let us assume binary antipodal transmission, i.e. $b_k \in \{+1, -1\}$, $1 \leq k \leq K$, and additive white Gaussian noise be the only distortion, and consider real path attenuations and signature sequences. Denote the tentative hard decisions of the auxiliary detector (and any temporary decisions in case of iterative detection, see Subsection 2.3) by $\hat{b}$ (this could be the hard quantized matched filter outputs (i.e., single user detector) or the outputs of a decorrelator [7, 8], for example), and denote the corresponding sequence deviating only in the $k$–th channel from $b = (b_1, \ldots, b_K)^T$ by $\tilde{b}_b$: $\tilde{b}_b = b_{k'} \forall k' \neq k$ and $\tilde{b}_b = -b_k$ if $k' = k$. Hence, the $K$ sequences $\tilde{b}_b$ are obtained by flipping successively just the $k$–th bit in $b$. Denote finally the likelihood difference by

$$\Delta_k = L(b) - L(\tilde{b}_b); \quad 1 \leq k \leq K,$$

which is a measure for the reliability of the $k$–th channel. Since (1) can be rewritten as

$$L(b) = \sum_{k=1}^{K} b_k \left(2y_k - \sum_{k'=1}^{K} c_k r_{k,k'} c_{k'} b_{k'}\right),$$

where $r_{k,k'}$ are the elements of the $K \times K$ cross–correlation matrix $R$, we obtain after substitution of (4) into (3)

$$\Delta_k = \hat{b}_k \left(y_k - \sum_{k'=1}^{K} c_k r_{k,k'} c_{k'} \hat{b}_{k'}\right);$$

$$\Delta_k = \Delta_k / 4; \quad r_{k,k'} = r_{k',k}; \quad 1 \leq k \leq K.$$

In some cases $\Delta_k$ will be negative, i.e., the complement of the tentative decision is decided to be more likely. Then it is advisable to rely on the new decision. Further generalizations such as iterative (multistage) detection or increasing of the search space (i.e., the chosen subset) will be discussed in Sections 2.3 and 3, respectively.

2.2 Analysis and Discussion of the $O(K)$ Likelihood Estimator

Equation (5) provides a simple rule to obtain reliability information. This algorithm is conceptually simpler than the DF multiuser detector proposed by Duel–Hallén [5, 6], because it does not need the spectral factorization and hence the matrix inversion to compute the feedforward matrix filter coefficients in [5, 6], which is $O(K^2)$ and complicates time–varying applications. (Once the coefficients of the feedforward matrix filter are computed, however, the number of operations is similar.)

We also found an indication that the error propagation phenomena seems to be less severe, which implies that an ordering of the channels would be less stringent.

Since the $k$–th matched filter output signal is given by

$$y_k = \sum_{k'=1}^{K} b_{k'} c_k r_{k,k'} c_{k'} + z_k; \quad 1 \leq k \leq K,$$

where $z_k = \int_0^T a_k(t) c_k n(t) \, dt$ is a colored Gaussian noise variate, we obtain from (5)

$$\Delta_k / \hat{b}_k = c_k^2 r_{k,k} + \sum_{k'=1; k' \neq k}^{K} c_k r_{k,k'} c_{k'} (b_{k'} - \hat{b}_{k'}) + z_k; \quad 1 \leq k \leq K.$$

This equation clearly indicates the decision–feedback mechanism inherent in (4). Since the term in brackets is zero when
the tentative decision is correct, or ±2 in case of an error, we finally obtain
\[
\Delta_k / b_k = c_k^2 r_{k,k} + \sum_{j: \text{error position}} c_j r_{j,k} c_k + z_k; \quad 1 \leq k \leq K,
\]
where \( j \) is (or are) the error position(s), \( 1 \leq j \leq K, j \neq k \). Errors in channels \( j \neq k \) in general affect the \( k \)-th channel if \( r_{j,k} \neq 0 \), which is the counterpart of error propagation in the DF multiuser detector.

From (7) and (8) we obtain that the conditional probability density function of \( \Delta_k \) is Gaussian distributed in case of no error propagation, and if the path attenuations \( c_k \) are time-invariant. Further analytical results, e.g., an evaluation of the probability density function of \( \Delta_k \) for the general case, an evaluation of the BER, or a proof that the likelihood differences \( \Delta_k \) are unbiased estimates, are not easy to obtain, since the noise varies \( z_k \) and errors of the tentative decisions are not independent, because of error propagation, and because the \( z_k \)'s are colored.

2.3 Iterative Detection with the O(K) Likelihood Detector

Among possible generalizations of the O(K) likelihood estimator is the following iterative scheme: We propose to run the O(K) likelihood detector iteratively until the signs of all \( K \) users do not change further. This operation needs to be started/maintained only when the new decision differs in at least one channel \( k \) from the previous temporary decision, i.e., if there exists at least one \( k \in \{1, \ldots, K\} \) such that \( \Delta_k < 0 \). Some modification might be required to prevent infinite alternation of signs of some (often unreliable) channels with each iteration. A variant is to declare erasures after a certain number of iterations for those channels where convergence is not obtained. Numerical results will be reported in Subsection 4.1.3.

3 Reduced-Complexity MLSD

The iterative scheme is a first attempt to tackle the error propagation problem. Another solution is based on the observation that the final BER is mostly affected by weak decisions, i.e., decisions in those channels \( k \) with a small likelihood difference \( \Delta_k \). The idea is to perform optimum detection only for those channels. The proposed reduced-complexity MLSD is as follows:

- Obtain reliability estimates, e.g., by running the O(K) likelihood detector, Eqn. (5), eventually with iterative refinement.

- Determine the \( L \) weakest reliability estimates, where \( 1 \leq L \leq K \) adjusts the complexity.

- Declare a final decision of the \( K - L \) most reliable decisions, \( \bar{b} \).

- Obtain a final decision \( \bar{b} \) for the remaining \( L \) channels by maximizing the likelihood function over the search space of all \( 2^L \) permutations given the \( K - L \) most reliable decisions \( \bar{b} \): \( \bar{b} = \arg \max \bar{b} \) \( L(\bar{b}_1, \ldots, \bar{b}_L, \bar{b}_{L+1}, \ldots, \bar{b}_K) \).

Given all final decisions, re-run (5) for the \( L \) channels \( k \) to obtain improved reliability estimates, if needed.

The MLSD is obtained for \( L = K \). Note that even the case \( L = 1 \) is superior than the O(K) likelihood detector (although only slightly), since the least reliable bit is flipped again given the (corrected) other decisions. In addition, an iterative refinement as discussed in Subsection 2.3 is feasible if \( L < K \). Numerical results are presented next.

4 Simulation Results

4.1 Example 1: Uncoded System, White Gaussian Noise

The first application under consideration is a synchronous DS-CDMA system with binary antipodal signaling shared by \( K = 4 \) active users which are identified by length \( N = 7 \) Gold sequences [4, 5]. The cross-correlation matrix, \( A \), is shown in Fig. 1. By inverting this matrix it is easy to see that user 1 and 4 are the weakest users if the received energy is the same for all users. The noise is additive white and Gaussian. Fig. 1 shows the BER of user 4 versus the signal-to-noise ratio of channels \( k = 1, 2, 3 \), which is varied between -4 dB and 14 dB. Channel \( k = 4 \) has a fixed signal-to-noise ratio of \( E_b/N_0 = 8 \) dB. (\( E_b \) is the signal energy per information bit and \( N_0 \) is the one-sided noise density.) We chose this scenario, because data for the conventional (single user) detector, the decorrelator and the DF multiuser detector are partially available [4, 5], and hence can be compared with our results, which further include the O(K) likelihood detector, reduced-complexity variants of the optimal detector, the optimal detector, and the single user bound. Perfect channel estimation is assumed for the DF detector, the MLSD, and our proposals.

Consider first the conventional detector: For \( E_b/N_0 \rightarrow 0 \), \( k = 1, 2, 3 \) (i.e., users 1, 2, 3 are not transmitted), the single user detector asymptotically approaches the single user bound for channel \( k = 4 \), because multiuser interference disappears (at the leftmost side of the figure). If \( E_b/N_0(k) \gg E_b/N_0(4), k = 1, 2, 3 \) (i.e., users 1, 2, 3 are error free), the BER of user 4 approaches 1/2 (at the rightmost side of the figure): This detector is not near-far resistant [7].

Next consider the decorrelator: This detector is near-far resistant, and its performance is independent of the signal power.

The decision-feedback (DF) multiuser detector, which is next, significantly outperforms the decorrelator, and asymptotically approaches the single user bound as multiuser interference disappears, i.e., \( E_b/N_0(k) \rightarrow 0 \), or as error propagation diminishes, i.e., \( E_b/N_0(k) \gg E_b/N_0(4), k = 1, 2, 3 \).

Since at some point users 1, 2, 3 are weaker than user 4 for

\( ^4\text{The ordering in (9) is only conceptual, differently to }[5, 6]: \text{Strictly speaking, the DF detector is not } O(K) \text{ if an ordering corresponding to the path attenuations is performed.} \)

\( ^5\text{This detector still is } O(K) \text{ since the minimum search is linear in } K. \)
If $E_b/N_0(k) < E_b/N_0(4)$, the DF is worse than the conventional detector due to error propagation. (We assume that user 1 is detected first, user 2 is detected second, etc.)

Of special interest is the O($K$) likelihood detector (tentative decisions of a decorrelator are improved by the O($K$) likelihood post--processor), which is slightly better than the DF (at least) in this example. No iterative refinement is conducted, hence the users can be detected in any order. (Again, we assume a detection order corresponding to the user indices to obtain a fair comparison with the DF.)

Finally, Fig. 1 shows the BER for the reduced complexity variants, $L = 1$ to 3, as well as the BER of the MLSD ($L = 4$). $L = 1$ basically tallies with the O($K$) detector, but the least reliable bit is flipped again given the (corrected) other decisions.

A more detailed comparison of the O($K$) detector and the DF detector is shown in Fig. 2. In both cases users are detected in increasing order. The likelihood detector is slightly superior.

Further investigations were conducted by running the iterative scheme presented in Subsection 2.3. Unless erasures are declared when convergence is not obtained after a certain number of iterations, the performance improvement obtained with this iterative scheme is weak. Results are close to the curve labeled $L = 2$.

4.2 Example 2: Coded System, Rayleigh Fading

In addition to the previous scenario, in this second example channel coding is applied and Rayleigh distributed path attenuations $c_k$, $k = 1, \ldots, 4$, are considered. Each user $k$ is independently encoded by a rate-1/2 4-state convolutional code. The encoded bits are sufficiently interleaved (almost completely) to randomize error bursts. The corresponding decoder is a soft--decision (=soft--input) Viterbi decoder with 32 information symbols delay, which is fed by the de--interleaved soft--outputs of the single or multiuser detector. All $c_k$ are mutually independent in $k$ and time, and are assumed to be perfectly known by the receiver.

Simulation results of this experiment (and for the uncoded case in addition) are presented in Fig. 3. In contrast to Fig. 1, the average BER is plotted versus the average signal--to--noise ratio per bit. Consider the uncoded case first: The decorrelator outperforms the conventional detector, but still degrades by about 2 dB from the single user bound at a BER of $10^{-1}$. This degradation is reduced to about 1 dB at a BER of $10^{-2}$ if post-processing is applied in addition (O($K$) detector). ML detection provides only slightly better results. For convenience, the remaining detectors investigated in the first experiment are dropped.

Results are similar in the coded case. The differences in dB are about the same: The gain of the O($K$) detector compared to conventional detection is on the order of 4 dB at a BER of $10^{-2}$, and the loss compared to a single user system is only about 1 dB at a BER of $10^{-2}$. This implies that a 'good' multiuser detector is essential, otherwise strong channel codes would be needed. But it implies too that truly combined multiuser detection/channel decoding is not needed from the performance/complexity ratio point of view.

'Good' multiuser detection, however, is not only characterized by the BER degradation, but also by the reliability of its soft--outputs if some kind of outer processing (such as channel decoding) is considered\textsuperscript{8}. The results in Fig. 3 are based on the utilization of channel state information (CSI): The outputs of the decorrelator\textsuperscript{9} were multiplied by the path attenuations $c_k$, $k = 1, \ldots, 4$, which is the optimum rule in the single user case. Conventional detector and post--processor, however, deliver CSI inherently, see (6) and (8) respectively, if the path attenuations, $c_k$, are included in the matched filter operation.

In addition to the presented results, simulations were also run without CSI: The system with conventional detection would further degrade by about 1 dB if $c_k$ in (6) would be dropped. If decorrelation would be performed without any post--processing, the results would be even worse than for conventional detection in the interesting range of BERs [11]. The reason for this surprising result is that the magnitude of the outputs is highly "misshaped" due to the zero--forcing effect. With decreasing noise, the estimated likelihood values asymptotically saturate (i.e., the magnitude of the outputs approach a constant). Similar results are expected for linear (especially zero--forcing) equalization.

5 Asynchronous DS--CDMA

So far we investigated the case of synchronous transmission only. The more general (and practical) asynchronous case can be handled as follows:

- Define the corresponding equivalent synchronous system [8, 5, 6].
- Substitute (1) by the likelihood function for the asynchronous case [1].
- Apply linear least--squares detection to generalize the decorrelator for the asynchronous case.

With this in mind, the bulk of this work generalizes straightforwardly. The effort will still be linear in $K$ (if $L \leq 1$). Since the equivalent synchronous system is blocked, only edge effects need to be investigated more carefully. Several possibilities apply, ordered by their performance:

- If the equivalent synchronous system is bounded by known training symbols, no side effects will occur.
- A bank of equivalent synchronous systems could be defined for all possible data symbols bounding the equivalent system.
- The edge symbols could be decided prior defining the equivalent system ("DF").

We expect that the main results will hold for the asynchronous case, too.

\textsuperscript{8}In case of DF multiuser detection the inputs to the slicers should be taken as the soft outputs.

\textsuperscript{9}The decorrelator works best with normalized sampled matched filter outputs, $\hat{y}_k = \int_0^T r(t) a_k(t) \, dt$ [8], instead with matched filter outputs including CSI as in (1).
6 Conclusions

Channel coding and multiuser detection for DS–CDMA channels is considered. Emphasis is on low-effort multiuser detectors, which provide reliability information to an outer channel decoder. The effect of channel state information is discussed.

A family of suboptimum multiuser detectors with adjustable performance/complexity tradeoff is proposed, which is derived from the MLSD in a structured manner. The basic idea to obtain reliability information is that likelihood evaluation and detection can be separated such that precomputed hard decisions are fed into the likelihood function, and only a subset of possible information bits is subject for an exhaustive search. The basic ideas to reduce the complexity are that the reliability difference is small for weak decisions, and that the reliability estimates in turn can be used to obtain a “good” subset. We further showed that the reliability generator can be viewed as a likelihood post–processor [11].

Two examples, for a Gaussian channel and for a coded Rayleigh fading channel, respectively, indicate that even the simplest version, whose effort is linear in the number of users $K$, performs reasonably well especially under near–far conditions (i.e., a large $K$ imposes no problem, and fading even ‘helps’). Its performance is similar to the DF multiuser detector defined in [5, 6], i.e., multiuser interference is basically compensated. Hence, truly combined multiuser detection/channel decoding is not needed from the performance/complexity ratio point of view, since the single user bound is approached within about 1 dB.

A final remark concerns the application: Post–processing is applicable to many other areas, e.g., linear equalization could be significantly enhanced both in terms of BER performance and soft–output capabilities. Also, reduced–complexity detection by searching only among a subset of unreliable decisions, is of more general interest.

References


Comparison of some Multiuser Detectors

N=7 Gold sequence, K=4 active user, AWGN channel

![Graph showing BER for the weakest user (user 4) vs. E_s/N_0(k) - E_s/N_0(4) dB, k=1,2,3, E_s/N_0(4)=8 dB.]

Figure 1: Simulation Results for Example 1.

Comparison of O(K) Detector with DF Detector

N=7 Gold sequence, K=4 active user, AWGN channel

![Graph showing BER vs. E_s/N_0(k) - E_s/N_0(4) dB, k=1,2,3, E_s/N_0(4)=8 dB.]

Figure 2: Simulation Results for Example 1.